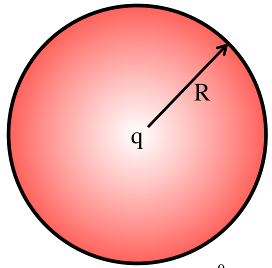
Problem 24.7

A hollow, insulating (non-conducting) sphere is centered on a charge *q*, where *q* is defined to the right below the sphere. A circular hole of radius .001 meters is drilled into the sphere. What is the flux through the hole?



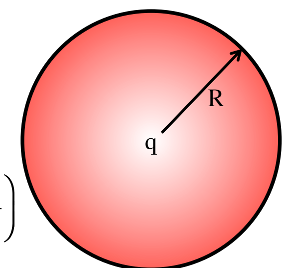
where $q = 10\text{x}10^{-9}\,\text{C}$
and $R = .1\text{ m}$

The total flux through the sphere will be:

$$\Phi_E = \frac{q_{enclosed}}{\varepsilon_o}$$

$$= \frac{(10\text{x}10^{-6}\text{C})}{8.85\text{x}10^{-12}\text{C}^2/\text{N}\bullet\text{m}^2} = 1.13\text{x}10^{6}\ \text{N}\bullet\text{m}^2/\text{C}$$

The flux through the hole will be a fraction of the whole such that:



where $q = 10\text{x}10^{-9}\,\text{C}$
and $R = .1\text{ m}$

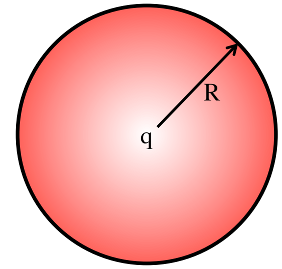
$$\Phi_E = (\Phi_{total})\left(\frac{A_{hole}}{A_{sphere}}\right) = (1.13\text{x}10^{6}\ \text{N}\bullet\text{m}^2/\text{C})\left(\frac{\cancel{\pi}r^2}{4\cancel{\pi}R^2}\right)$$

$$= (1.13\text{x}10^{6}\ \text{N}\bullet\text{m}^2/\text{C})\left(\frac{(.001\text{ m})^2}{(.1\text{ m})^2}\right)$$

$$= 28.25\ \text{N}\bullet\text{m}^2/\text{C}$$

ADDITIONAL STUFF:

Interesting, if we used Gauss's Law to determine the electric field, evaluated at the sphere's surface, we could then multiply that value by the area of the hole to determine the flux through the hole. As that will allow us to mess with Gauss's Law, let's give it a try. First, a summary:



1.) Gauss's Law require you to evaluate the left and right side of the following equation:

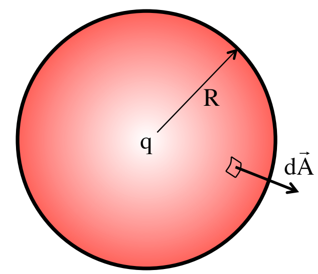
$$\int_{sa} \vec{E}\bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

2.) The action on the right side is quite literally the sum of the charges, signs included, residing within the Gaussian surface. Determine that quantity is usually the hardest part of the problem.

3.) The left side is essentially a dot product. Worked out, it looks like:

$$\int_{sa} |\vec{E}||d\vec{A}|\cos\theta$$

4.) There are a couple of things to notice about this dot product.



$$\int_{sa} \vec{E}\bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\int E\,dA\cos$$

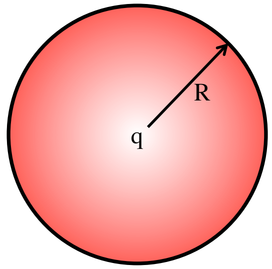
a.) The differential area vector *dA* is ALWAYS directed outward, relative to the surface, in the + radial direction.

b.) In theory, the electric field's direction should be oriented outward in the + radial direction if the net charge inside the Gaussian surface is positive, and in the – radial direction if the net charge inside the Gaussian surface is negative.

c.) The problem is that when you do a complex version of one of these problems using variables instead of numbers (this, I might add, is the norm), you can't tell whether the net charge inside the Gaussian surface is positive or negative.

d.) The way to deal with this is to always assume the direction of the E field is outward in the + radial direction, which means that the dot product and, hence, the left side of the equation, will ALWAYS be positive.

e.) The electric field is supposed to be a magnitude, which is to say *positive*. The beauty of the assumption suggested in *Part d* is that if you keep accurate track of the sign of the charges inside the Gaussian surface (i.e., the right side of the equation) and you end up with a net sign that's negative, all that sign is telling you is that your assumed direction of the electric field (which was positive and outward) was wrong, and it's direction is opposite that. In short, you will never have to divine the direction of the electric field so as to get the right angle between E and dA for the dot product. Cool, yes?

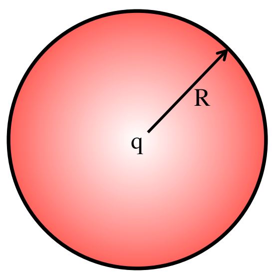


$$\int_{sa} \vec{E}\bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\int E\,dA\cos$$

5.) The left side of the relationship (the dot product) will, for a given geometry, ALWAYS have the same general form. Our problem's Gaussian surface assumes a *spherical geometry*, so that is what we will take up first.

6.) Noting that this is one of the rare occasions when the right side is easy, we can write:



$$\int_{sa} \vec{E}\bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\Rightarrow \int_S |\vec{E}||d\vec{A}|\cos 0^\circ = \frac{q}{\varepsilon_o}$$

Due to symmetry, the magnitude of E is the same everywhere on the Gaussian surface. Being constant, we can pull it out of the integral leaving:

$$|\vec{E}|\int_S |d\vec{A}| = \frac{q}{\varepsilon_o}$$

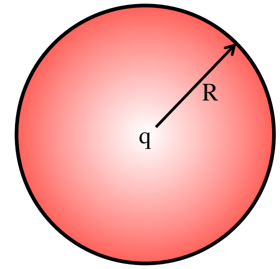
The sum of all the differential surface areas over the entire surface is just equal to the surface area of the Gaussian sphere. That means:

$$|\vec{E}|\int_S |d\vec{A}| = \frac{q}{\varepsilon_o}$$

$$\Rightarrow |\vec{E}|\left(4\pi R^2\right) = \frac{q}{\varepsilon_o}$$

$$\Rightarrow |\vec{E}| = \frac{q}{\varepsilon_o\left(4\pi R^2\right)}, \text{ or}$$

$$\Rightarrow |\vec{E}| = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2}.$$



And shazam, we've just derived the electric field function for a point charge.

Multiplying that by the area of the hole and we have the flux through the hole!

$$\Phi_{hole} = |\vec{E}|A_{hole} = \left(\frac{1}{4\pi\varepsilon_o}\frac{q}{R^2}\right)\left(\pi r^2\right)$$

$$= \left(9x10^9\right)\left(\frac{10x10^{-6}C}{(.1\text{ m})^2}\right)\left(\pi(.001\text{ m})^2\right) = 28.27\text{ N}\bullet\text{m}$$

Q.E.D.